Logic Seminar 781-Organizational Meeting this Thursday 2:55, Malott 230.

If you have topics you wish to present, bring them along. I would like as general but not exclusive theme:**Recursive Realizabilities of Constructive Logics**.

Recursive realizabilities, anticipated by Heyting and first introduced by Kleene, may be regarded as (intensional) functional interpretations of intuitionistic mathematics. That is, when (for all x)(there exists a y)... is provable, then from an index of a recursive x, you can compute an index of a recursive y, etc. I want to go through this literature as a possible general methodology for independence proofs in constructive and recursive algebra and analysis. Under any reasonable realizability, recursive or not, all theorems are realized. Statements that are not realized are not theorems This is a method for showing that constructive proofs (at least within the constructive systems employed) are not possible. Everyone knows that recursive realizability can be used to show that what is constructively provable is recursively correct. The game I am interested in is how to design realizabilities to show that a given statement cannot be realized. This is intimately related to recursive forcing. One can also use this to give a constructive version of reverse mathematics, tho there is nothing in te literature on this topic at present.

Realizability semantics is a natural alternative to Kripke model semantics for intuitionistic systems. It is quite likely that the most appropriate model theory for intuitionistic mathematics is such interpretations, since they are by nature intensional, referring to indices of functions rather than the functions themselves. These can also be thought of as applied lambda calculus interpretations. I would like to proceed in historical order. First have someone do realizability for arithmetic and analysis from Kleene's two books and Beeson's book, then Troelstra's higher order realizability, then Girard's intuitionistic higher order logic realizability, then McCarty's and Scedov's work in intuitionistic set theory, then Dana Scott's school's work in the topos and general lambda calculi, then a little Martin-Lof anda Constable, and Huet-Coquand. There was a European seminar organized by the Scott group at Carnegie Mellon this summer on realizability. This seems to be mostly topos-theoretic, sheaf theoretic, domain-theoretic. The bibliography they produced is at

http://www.cs.cmu.edu/~birkedal/realizability-bib.html

Some papers will be available on the web at the end of August,
which will be announced at http://www.cs.cmu.edu/Groups/LTC/ at
the end of August.

There is also a recent paper of Troelstra in the Handbook of
Proof Theory edited by Sam Buss.