Math 513 Topics in Analysis MWF 10:10-11:00

White Hall 310

I will give a course presenting some basic methods and results of geometric PDE. The stress will be to obtain results and estimates for linear and nonlinear elliptic equations in a general geometric setting with only natural geometric hypotheses. Topics will include comparison theorems for the distance function and consequences, Harnack-type estimates for solution of the Laplace and heat equations. Some general results concerning the spectrum of manifolds as well as geometric inequalities such as Poincaré and Sobolev inequalities.

José F. Escobar

Math 519, Fall 1994

Partial Differential Equations (PDE)

MWF 12:20-1:10

WHITE 310

This is a first course in PDE for graduate students in Mathematics, and for students in other subjects with sufficient background in Mathematics. No previous experience with PDE's is assumed.

We shall treat in depth model problems from the three main classical types of second order PDE's:

Hyperbolic (the wave equation)
Elliptic (Poisson's equation)
Parabolic (the heat equation)

The investigations will be "classical" but based on the theory of distributions. (Hilbert space methods are left for 520.) You will need a background in analysis à la Math 413-414 here at Cornell. (If you haven't seen the Lebesgue integral you will have to take a few things on faith.)

The text for the course is

V. S. Vladimirov, Equations of Mathematics Physics, Marcel Dekker 1971.

There is an expanded version with the same title, published by Mir 1984. Both of those books are on reserve in the Mathematics Library.

Lars B. Wahlbin
White B-10

MATH 549: INFINITE DIMENSIONAL LIE ALGEBRAS (F. Akman)

T R 11:40-12:55 B25 White Hall

Math 549 will be an introduction to finite dimensional and graded infinite dimensional (tame) Lie algebras, and no prior acquaintance will be assumed. Lie algebras are central objects in both mathematics and physics but are overshadowed by Lie groups in standard graduate education. It is usual to think of a Lie group as the primary -geometrical or abstract- symmetry object (e.g. the rotation group as preserving the physical properties of some system), and its Lie algebra as secondary. However, Lie algebras have the advantage of being very simple and linear, and one might as well acknowledge their existence instead of resorting to phrases like "infinitesimal rotations" all the time. In any case, with some restrictions, (finite dimensional) Lie groups and Lie algebras are in one to one correspondence and have the same representations.

Due to time limitations and personal bias, I will work towards establishing an elementary knowledge of the representation theory of infinite dimensional graded Lie algebras (like Kac-Moody and Virasoro), which has become a prerequisite to reading literature in certain areas of mathematical physics, such as conformal field theory. It is my hope that on the way there will be a lot of avenues to be explored for the non-specialist, as every topic will be developed from scratch. There will be occasional references to differential geometry and physics but the principal approach will be algebraic.

Here is a rough and optimistic sketch of what is to be covered: Introduction to Lie algebras and their representations (derivations, tensor products, universal enveloping algebras...); finite dimensional Lie algebras (theorems on semisimplicity, classification of complex simple Lie algebras...); infinite dimensional "tame" Lie algebras (completions of enveloping algebras, normal ordering, category O, highest weight representations...); vertex operator superalgebras (VOSA's), or chiral superalgebras, as representation spaces of tame Lie algebras (generalizing the supercommutative associative algebras on which a Lie algebra acts by derivations); classical homology and cohomology of finite and infinite dimensional Lie algebras, a unified approach with de Rham cohomology; semi-infinite cohomology of tame Lie algebras; VOSA's as semi-infinite cohomology (e.g. BRST) complexes.

Math 552, Fall Semester, 1994-95
Differential Geometry

John Hubbard

Part 1: Curves and surfaces.

We will start with curves and surfaces in $\mathbf{R}^3$. I will cover this material more thoroughly than is usual.

Topics will include:

The Frenet frame, curvature and torsion of space curves;

Milnor's theorem on the total curvature of knots (see Spivak's lecture in the Milnor Symposium);

The second fundamental form of surfaces in space;

The Gauss-Bonnet formula for embedded surfaces;

Geodesics on surfaces, the first and second variation formulas;

The relation between Jacobi's equation and Sturm-Liouville theory;

The Theorema Egregium of Gauss.

All of this should take about 1 month. I have notes for the surface material which I will distribute.

Part 2: Manifolds, forms, etc.

In this part we will build up some of the essential material. In particular, we will prove

The convergence of Newton's method and the implicit function theorem;

Sard's theorem (Milnor's book *Topology from a Differentiable Viewpoint* is a good reference);

Existence of partitions of unity;

This will allow us to construct differential manifolds and their tangent bundles. We will then define:

Differential forms and their integrals, (Spivak's book *Calculus on Manifolds* is a nice reference)

Stokes theorem and the Poincaré lemma. (Again, I have notes)

Just how long this will take will depend very much on the audience. It might be 2 weeks, or it might be 5 weeks.

Part 3: Riemann Metrics, Connections, Curvature, Characteristic Classes

In this section we will introduce vector bundles and connections on these. We will define the Levi-Civita connection associated to a metric, and perhaps the complex analog. The curvature of a connection comes next, and the characteristic forms as polynomials in the curvature forms.

Other material will depend on time and the audience: Yang-Mills fields, hyperbolic geometry in 3 dimensions, complex differential geometry are possibilies.

Math 561 Geometric Topology   MWF 10:10   B-25

This is intended to be an introduction, at a fairly low level, to some of the more geometric aspects of Topology, mainly in dimensions two and three. There will be three main focal points, all related to each other: Surfaces, Knot Theory, and 3-Manifolds. One of the beauties of this subject is that there are so many interesting and nontrivial concrete examples which one can see rather explicitly, and these will play a large role in the course. For example, the course will begin with a study of the various ways in which the group $SL_2(Z)$ appears in low-dimensional topology, as a simple model for more difficult phenomena.

Prerequisites for this course are fairly minimal: some knowledge of fundamental groups, covering spaces, and basic homology theory as in 551, plus a modest acquaintance with manifolds, e.g., tangent bundles, as in 552 (but no differential geometry will be needed).

Allen Hatcher

Math 571: Probability Theory

The first part of a two semester introduction to probability theory, taught from my book **Probability: Theory and Examples**. Students are assumed to be familiar with measure theory as taught in 511 or 521, but all the necessary facts are contained in an appendix to the book. We will cover most of the unstarred sections in Chapters 1–3. In terms of topics this means

Basic definitions: random variables, expected value, independence

Weak and strong laws of large numbers

Weak convergence, characteristic functions, central limit theorem

Basic properties of random walks: Wald's equation, recurrence

If you have any questions about the course or your readiness for it, see me or send me e–mail: durrett @ math . cornell .edu

Rick Durrett

COURSE ANNOUNCEMENT

Mathematics 617 / T&AM 776: Applied Dynamical Systems

Fall 1994

John Guckenheimer

The book Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields by Guckenheimer and Holmes grew from notes for this course and will be used as a text. Topics will include

- Phase portraits for two dimensional vector fields
- Structural stability and bifurcation of two dimensional vector fields
- Center manifolds and normal forms
- Averaging theorem and perturbation methods
- Melnikov's method
- Discrete dynamical systems
- Smale horseshoe and other complex invariant sets
- Global bifurcations, strange attractors and chaos
- Applications

The lectures will include some new material from research done during the past decade. Computational methods and exercises will be stressed.

Mathematics 635 Course Description.

Topics in Commutative Algebra

Mark Gross

This course will be a sequel to Math 534 taught in the Spring of 1994. I intend to cover a good deal of further material from David Eisenbud's book *Commutative Algebra with a view towards Algebraic Geometry*, or Matsumura's book *Commutative Ring Theory.* One basic goal is to cover all the commutative algebra results used in Hartshorne's *Algebraic Geometry*, but we should have plenty of time for topics past this. Topics may include the following: (Some of these were listed in the 534 Syllabus)

(1) Flat modules.

(2) Modules of differentials.

(3) The theory of depth.

(4) Cohen-Macaulay, Gorenstein, and regular rings.

(5) Finite ring extensions, étale ring extensions, Henselian rings.

(6) Homological techniques:

> Regular sequences, the Auslander-Buchsbaum theorem, local cohomology, local duality. (This includes a lot of material)

(7) Gröbner bases.

I will definitely assume some familiarity with homological algebra, but will try to review as much of this as is possible. I intend to give many applications to both Algebraic Geometry and Number Theory.

Math 637 Number theory

This course is an introduction to automorphic forms on adelic groups and Jacquet-Langlands theory. The following topics will be covered

1. Clasical L-functions, their functional equation and product formula
2. Classical automorphic forms and their relationship to representations of of the general linear group GL(2,$\mathbf{R}$)
3. p-adic numbers, adels and L-functions of Hecke characters
4. Representations of the general linear group over the adels and their relationship to classical automorphic forms
5. Representations of the general liner group in 2 variables over real, complex and p-adic fields and their L-functions
6. Representations of the general linear group over the adels and their L-functions

Birgit Speh

Math 651

BERSTEIN SEMINAR IN TOPOLOGY

MWF 11:15 310 White Hall

This course is a seminar in which students explore a single topic or series of related topics in topology. The students present the material to the rest of the class, with questions and discussion actively encouraged.

The topic I would like to focus on this semester is the study of the group of automorphisms (or outer automorphisms) of a free group, in particular what can be learned from considering the action of this group on Outer space. There are analogies between this action and the action of an arithmetic group on a homogeneous space, or the action of the mapping class group of a surface on the Teichmüller space of the surface. The exact choice of topics is ultimately up to the students who will present the material, and will be discussed in more detail at the first class meeting.

Karen Vogtmann

Math 657

Controlled Simple-Homotopy Theory and Manifold Topology

INSTRUCTOR: PAUL THURSTON

DAY AND TIME: TR 8:40 A.M. HO 206

Fall Semester 1994

The intent of this course is to provide the student with an deep conceptual understanding of the key elements of *controlled manifold topology.* Our investigation of this topic is inspired by the recent flurry of activity in the field and it is anticipated that many fruitful investigations are to be had in the future.

The course will follow weave its way through the modern literature (some of which only exists in pre-print form). The intent will not be to establish the most general statement of known theorems, but, on the contrary, to thoroughly understand enlightening (and applicable) special cases. Our aim is 3-fold.

I. Controlled simple-homotopy theory and Controlled h-cobordism theorems

Our initial concern will be to establish the bounded and controlled versions of the h-cobordism and s-cobordism theorems, which form the backbone of this study. We will find ourselves working on both topological (controlled handle calculus) and algebraic (negative K-theory) aspects of the topic.

II. Ends of Maps and a Controlled Splitting Theorem

Building from the controlled h-cobordism theorem, our next task is to establish *Quinn's Ends Theorem* and as well as a *controlled splitting theorem.* Essentially a controlled version of *Siebenmann's thesis*, the ends theorem is the starting point for *resolution obstruction theory,* as well as a number of modern developments. The controlled splitting theorem is a key technical ingredient in a number of controlled surgery settings.

III. Bounded Surgery and Applications

Finally, we'll introduce the notion of surgery with bounded control and establish the exactness of the bounded surgery exact sequence. With the last of the three elements in place, we'll be able to make quick work of a few of the notable results in geometric topology, including the famous *Annulus and Stable Homeomorphism Conjectures*, the *Product Structure Theorem* and *Siebenmann's Cell-like Approximation Theorem.*

Prerequisites: 2 semesters of algebraic topology and familiarity with either PL or $DIFF$ manifolds (i.e. regular/tubular neighborhood theory) or consent of the instructor.

Hyperbolic Geometry and Kleinian Groups

The 3-Manifold Hyperbolization Theorem

Frédéric Paulin

MATH 661, Mon-Wed 3:35-4:50, Room : WE B29

The aim of this course is to give a fairly complete proof of W. Thurston's hyperbolization theorem for 3-manifolds, using the new approach of C. McMullen (non fibered case) and J.-P. Otal (fibered case). This will include (obviously without complete proofs !) the necessary backgrounds in

- Hyperbolic geometry (geometry of the convex hull, Margoulis lemma)
- Quasiconformal maps and Teichmuller theory (complex structure of the Teichmuller space)
- 3-dimensional topology (from Dehn's lemma to Waldhausen's finiteness theorem)
- Kleinian groups (dynamics of the limit set, geometrically finite Kleinian groups, Ahlfors' measure zero theorem, Klein-Maskit's combination theorem)
- Pseudo-Anosov diffeomorphisms and geodesic laminations on hyperbolic surfaces (Thurston-Nielsen theory of automorphisms of surfaces)
- Group actions on $\mathbf{R}$-trees (and Skora's theorem).

We will then give a complete proof of Thurston's orbifold trick, C. McMullen's proof of the contraction of the Θ-operator and J.-P. Otal's proof of Thurston's double limit theorem using degeneration of hyperbolic structures to $\mathbf{R}$-trees. The hyperbolization theorem will then follows.

Prerequisite : Basic courses in Differential geometry, Complex analysis, Algebraic Topology. Some knowledge on low dimensional topology would help. References to be given during first course.

Math 667 **Algebraic Geometry: Curves and Surfaces**
Stillman, Fall 1994

In this course we will study algebraic curves and surfaces, roughly at the level of chapter four and five in Hartshorne's book. Examples will be emphasized, and computational methods will be discussed. However, this is not just a computational course.

My goal for this course is to teach the basic theory and examples of curves and surfaces, and to teach the use of some of the more abstract techniques (e.g. sheaf cohomology) in a very concrete setting.

In the first half of the course, the topics will be

– Hilbert functions and Hilbert polynomials.

– Projections, blow ups, tangent cones, multiplicity.

– Smooth models of curves.

– Differentials, branch loci, Riemann-Hurwitz theorem.

– Divisors, and then Riemann-Roch for curves, with emphasis on many applications, examples.

– maps to projective space, including canonical curves, curves in $\mathbf{P}^3$, and mapping curves into the projective plane, secant loci.

– examples: rational normal curves, elliptic curves, hyperelliptic curves, and curves of genus less than 5 or 6.

– Weil Divisors, Cartier divisors, line bundles, linear systems, maps to projective space and how to compute with them.

After that, we will cover surfaces, basically using Hartshorne's book, chapter 5, as a model. Along the way, we will introduce sheaf cohomology when it becomes useful.

Meeting time and place: White Hall 310, Tuesday, Thursday 10:10 – 11:25.

Textbook: Hartshorne: Algebraic Geometry.

Recommended:

Harris: Algebraic Geometry: A First course (Springer).

Mumford: Algebraic Geometry I: Complex projective varieties (Springer).

Homework: It is not possible to learn algebraic geometry without working alot of problems and understanding alot of examples. We will give problem sets every two weeks or so, and then discuss the solutions in a separate class meeting.

Prerequisites: Chapter one of Hartshorne, and some commutative algebra, roughly at the level of Math 534. Also, knowing some sheaf theory would help, although we will review this material when we need it. Any results from Chapters 2 and 3 of Hartshorne that we need, we will at least state precisely.

MATH 677 FALL 1994

I plan to teach a course on STOCHASTIC INTEGRATION AND STOCHASTIC DIFFERENTIAL EQUATIONS. The course will probably cover only the fundamentals of the subject since I am far from being an expert in this area. However, the level and choice of topics is flexible and will depend on the audience.

H. Kesten

MATH 684
RECURSION THEORY
FALL 1994

This term math 684 will concentrate on the structure of the Turing degrees as a whole (i. e. of all sets). The text will be *Degrees of Unsolvability* by M. Lerman, Springer–Verlag. We will assume some background in logic and at least a minimal acquaintance with Turing machines or some other model of computation. Math 581 or Computer Science 682 should be more than sufficient.

We will begin with a brief discussion of the basic properties of a reasonable model of computability: universal machines, the enumeration, s–m–n and recursion theorems, r. e. sets and the halting problem (one week). Next will come the notions of relative computability, the Turing jump operator and the arithmetical hierarchy (two weeks). There will be some discussion of recursively enumerable sets and degrees with at least one or two priority arguments. The heart of the course, however, will be the analysis of of various kinds of priority arguments for the construction of r. e. sets. We will first study the finite–extension constructions of Kleene and Post and the related notion of forcing in arithmetic (two weeks). These methods will be extended to other notions of forcing to give the Slaman–Woodin proof that the theory of the Turing degrees is as complicated as possible: it is recursively isomorphic to the theory of true second order arithmetic. We will also introduce perfect set forcing approximations to construct a minimal degree and then extend the methods to embed large classes of lattices as initial segments of the degrees. We hope to discuss some of the more recent results restricting possible automorphisms of the degrees and related homogeneity questions.

Richard A. Shore

Mon 4:30
B25

APPLIED LOGIC
Math 688
Fall, 1994

Richard Platek

August 25, 1994

Topic: Non-Standard Analysis

Instructor: Richard Platek

Time: The course is currently scheduled to meet TR 4:25-5:40, WE 310. This may conflict with some schedules so we will hold our first meeting 25 August at 4:25 and decide whether we should move the time.

Non-standard analysis (NSA) is an approach to analysis which uses sufficiently "saturated" (non standard) models of set theory to replace limit arguments. Intuitively, "saturated" means that anything that is consistent to add to the universe has already been added. NSA was originally introduced by the logician Abraham Robinson in order to provide a denotational semantics for those pre-nineteenth century formalisms in which the calculus was developed using actual infinitesimals and infinities. Robinson's approach is consistent with the dominant historical trend in mathematics which justifies formal manipulations by extending the ontology (from naturals to rationals to reals to complex numbers, etc.). In this manner Robinson was able to give rather amazing, very short, intuitively clear proofs for much of the classical calculus of one and several variables. NSA provides a more logical, algebraic or discrete approach to the whole body of mathematics which is usually developed using limits. Post-Robinson NSA extends these methods to more advanced, graduate school level, analysis (e.g., seemingly magical proofs of the existence of measures; more generally, the development of infinitary probability theory from finite, combinatorial probability theory through the use of infinite natural numbers). NSA is a true delight. It should appeal to logicians or algebraists who always wanted to know more analysis but couldn't quite master the epsilon-delta business; analyists who are either interested in the foundations of their subject or who would like to learn an elegant new method to add to their arsenal; applied mathematicians who feel comfortable with infinitesimals and never bothered to learn that there is something wrong with them; general mathematicians with an historical sense who enjoy seeing Euler, et al justified; computer scientists interested in hybrid systems as well as a method which yields more logical, non-numerical analysis correctness proofs for mathematical software.

All necessary logical and analytical background will be developed as needed.