MATH 511, FALL 1996
MEASURE THEORY

The textbook for the course is W. Rudin, Real and Complex Analysis, 3rd edition. We hope to cover most of Ch. 1-9. The titles of these chapters are as follows:

Abstract Integration

Positive Borel Measures

L^p-spaces

Elementary Hilbert Space Theory

Examples of Banach Space Techniques

Complex Measures

Differentiation

Integration on Product Spaces

Fourier Transforms

Mathematics 513
Topics in Analysis
Holomorphic dynamics on the Riemann sphere:
rational maps and Kleinian groups
Fall 1996
Dr. Kevin M. Pilgrim

This will be an introductory-level graduate course; the prerequisites include only a good course in complex analysis and some knowledge of covering spaces.

A *rational map* is a holomorphic map of the Riemann sphere to itself; a *Kleinian group* is a discrete group of holomorphic automorphisms of the Riemann sphere. Kleinian groups were first studied systematically by Poincaré, and have been studied intensively ever since as they arise in many diverse areas of mathematics, from number theory to three-dimensional topology. The study of the dynamics, or behavior under iteration, of a rational map was first begun by Fatou and Julia in the 1920's. In the 1970's computer-generated pictures of Julia sets, the fractals arising from iterating a rational map, rekindled interest in the subject, and it has since grown into a rich and beautiful field which is now actively studied from many different points of view. In the 1980's came the observation by Sullivan that the theories of Kleinian groups and rational maps are remarkably similar.

The goal of this course is to develop in parallel the theories of rational maps and Kleinian groups, viewed as holomorphic dynamical systems on the Riemann sphere, emphasizing those techniques and results which are basic (and strikingly similar) to both fields.

There will be two or three homework sets and a short writing project required. There will be no text *per se* but we will draw upon J. Milnor's paper (IMS Stony Brook Preprint No. 1990/5) for material on rational maps.

Content of the course:

O. Overview

1. dynamical systems in general
2. rational maps–examples and some history
3. Kleinian groups–examples and some history

I. Background in analysis

1. covering spaces
2. uniformization theorems
3. classification of automorphisms of the sphere, plane, and disc
4. the hyperbolic metric

5. the Schwarz Lemma and its generalizations
6. Carathéodory's theorem and local connectivity
7. univalent functions and Koebe distortion
8. Montel's theorem and normal families of holomophic functions

II. Rational maps and Kleinian groups

1. examples
2. Fatou sets and domains of discontinuity
3. Julia sets and limit sets
4. general properties
5. dynamics on the domain of discontinuity: Ahlfors-Bers finite area theorem
6. dynamics on the Fatou sets: classification of stable regions and Sullivan's No Wandering Domains theorem

III. Expanding dynamics

1. equivalence of various definitions
2. area, local connectivity, and Hausdorff dimension of Julia sets and limit sets
3. combinatorial models for Julia and limit sets

IV. Parameter spaces

1. conjugacies
2. structural stability
3. rigidity and some good thesis problems

Math 519, Partial Differential Equations, Fall 96

Teacher: Professor Vladimir Veselov

Text: Fritz John, Partial Differential Equations, fourth edition, Springer Verlag.

This is a basic course on the theory of partial differential equations (PDE). It will not assume any prior familiarity with PDE, but assumes a background in Analysis and some familarity with ordinary differential equations. Basic theory of first order PDE and linear PDE of second order will be covered.

A natural sequel is Math 520 in the Spring, which will cover Hilbert space theory of PDE.

ALGEBRA 531

Fall 1996

MWF 10:00–11:00am, WE 310.

This is a basic graduate course in Algebra. The prerequisites are a basic undergraduate knowledge of abstract algebra and linear algebra. Here is an outline of topics, the material can be found in many standard textbooks.

- Groups: cyclic groups, normal subgroups, Sylow's theorems, composition series, the permutation group.
- Rings and Algebras: modules, exact sequences, tensor products, PIDs, UFDs.
- Galois Theory: fields, field extensions, Galois groups, solvability of equations by radicals.

Grading. There will be weekly homework assignments and a take–home final. Possibly there will be a midterm take–home; or I might ask you to give talks on some topics instead of an exam. Generally, homework will be handed out each Monday and collected a week later.

REFERENCE BOOKS

Hungerford, *Algebra.*
Lang, *Algebra.*
N. Jacobson, *Basic Algebra.*

Dan Barbasch
226 White Hall
255–3685
barbasch@math.cornell.edu

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Math. 549 –Fall 1996

LIE ALGEBRAS/REPRESENTATION THEORY

Lie Algebras, Lie groups, and their representations play an important role in much of Mathematics, particularly in Combinatorics, Mathematical Physics and Topology.

This is a basic course in Lie algebras and their Representation Theory. The prerequisites are a basic knowledge of algebra, linear algebra. The course should be as self contained as possible. I will be trying to cover the following topics.

- Definition of Lie algebras; nilpotent, solvable and reductive Lie algebras; basic structure theorems.
- Semisimple Lie algebras; Root systems and their Classification;
- Enveloping Algebras and PBW-theorem;
- Representation Theory of semisimple Lie algebras.

If time permits, we will discuss Kac-Moody algebras and Quantum Groups.

REFERENCE BOOKS

J. Humphreys, *Introduction to Lie Algebras and Representation Theory.*
J. Dixmier, *Enveloping Algebras.*
W. Fulton and J. Harris, *Representation Theory. A first course.*

Arkady Berenstein
420 White Hall
255–4959

MW 2:55-4:10, WHITE B-25

MATHEMATICS 552 FALL 1996

(First semester of a 2-semester sequence Math 552 – 553)†

DIFFERENTIABLE MANIFOLDS

This is an introductory course aimed at first year graduate students. Here is a tentative schedule:

1. TOPOLOGICAL MANIFOLDS (2 weeks)

Basic point set topology of manifolds; boundary and interior — the consequences of Brouwer's theorem on Invariance of Domain (which is taught in Math 551); homogeneity of manifolds.

2. DIFFERENTIABLE MANIFOLDS (8 weeks)

Differentiable ($= C^\infty$) structures on topological manifolds; the implicit function theorem and submanifolds of smooth manifolds; smooth embedding of a manifold in $\mathbf{R}^N$; the tangent bundle (and vector bundles in general); vector fields; the Lie bracket; differential forms; connections and parallel transport.

3. RIEMANNIAN MANIFOLDS (4 weeks)

Riemannian metrics; Riemannian connections; geodesics; Riemannian curvature; lots of fun with Christoffel symbols.

VIEWPOINT: This is a subject in which the beginner can be overwhelmed with notation. I hope to keep things in perspective with a strongly historical viewpoint and with continual emphasis on basic ways that examples arise: submanifolds of $\mathbf{R}^n$, quotients of Euclidean, hyperbolic and spherical n-space by smooth or isometric actions of groups, Lie groups.

PREREQUISITES: Undergraduate topology ($\sim$ Math 453), linear algebra ($\sim$ Math 433) and a strong background in advanced calculus of several variables.

MOST LIKELY TEXT AS OF SPRING '96: Boothby, *An Introduction to Differential Manifolds and Riemannian Geometry*, Second Edition, Academic Press, 1986. The text will only serve as a rough guide and reference.

Marshall Cohen
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marshall@math.cornell.edu

† This program will probably run over a few weeks into the spring semester, so that the topics above can be done with some care and enjoyment. The main part of Spring '97 will then be spent on topics in Differential Topology (rather than Geometry): transversality, handlebody theory, and the h-cobordism theorem are likely topics.

Math 571: Probability Theory

Fall, 1996

M571-M572 is a one-year graduate sequence course on Probability Theory. We will use Rick Durrett's "*Probability: Theory and Examples, Second Edition*" as textbook. The topics of M571 will include

Probability sample spaces, random variables, distribution functions;

Expectation and moments,

Independence,

Weak and strong laws of Large Numbers,

Weak convergence, characteristic functions,

Central Limit Theorems,

Random Walks, stopping times, transience and recurrence, arcsin laws.

The prerequisite for this course is a knowledge of measure theory and Lebesgue integration, such as covered in Math 521, or Math 413-414.

Other Reference Books

Leo Breiman: Probability. 1968.

William Feller: An Introduction to Probability theory and its Applications. Vol. I, third edition (1968); Vol II, second edition (1971).

Zhen-Qing Chen

MATHEMATICS 611
SEMINAR IN ANALYSIS
Quasiconformal geometry and Teichmuller theory
FALL 1996

This course is an introduction to quasiconformal mappings and Teichmuller theory. We will cover the following topics:

1. Quasiconformal Mappings
   - Geometric and analytic definitions
   - Beltrami equation
   - Boundary dilatation

2. Teichmuller Spaces of Plane Domains
   - Teichmuller's metric
   - Complex structure
   - Extremality

3. Quadratic differentials
   - Teichmuller's mappings
   - Infinitesimal theory
   - Weak convexity

4. Some applications in Dynamics
   - Holomorphic motions and vector fields on closed sets
   - Teichmuller spaces of the complement of a Cantor set

References:

- L. V. Ahlfors, Lectures on quasiconformal mappings.
- F. P. Gardiner, Teichmuller theory and quadratic differentials.

Nikola Lakic
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Note: room change:
White 328
TR 2:55 - 4:10

COURSE SYLLABUS

Mathematics 617 and T&AM 776: Applied Dynamical Systems
Fall 1996

John Guckenheimer
Rhodes Hall 456
Tuesday-Thursday 1:25-2:40

Prerequisistes: Good courses in undergraduate analysis, multivariable calculus and linear algebra. Some exposure to ordinary differential equations or dynamical systems will be helpful.

The dynamical systems studied in this course have three characteristics:

1. The state of the system is a point of a finite dimensional manifold.

2. The evolution of the system is deterministic.

3. The equations defining the system are differentiable.

A large body of material has developed over the past thirty years to describe the qualitiative behavior of "generic" systems. Much of this course will be devoted to this theory, with special emphasis given to examples, applications and the role of computation in bringing the theory to bear upon examples. Large parts of the course will deal with bifurcation theory: how the qualitative properties of systems change in varying parameters. Some time will be spent developing background material from differential topology and singularity theory. Numerical methods for applying the ideas discussed to examples will be discussed.

Requirements: Homework and a take home final exam or a student selected project.

Math 631
Prof. Ehrenborg
Fall 1996

Topics in Algebra
Topics in algebraic combinatorics

We will give a flavor of algebraic combinatorics, that is, algebra applied to combinatorial problems. Topics will mainly be focused on posets and their properties.

- Posets and their incidence algebra.
- The Möbius function and how to compute it.
- Eulerian posets and the generalized Dehn-Sommerville equations.
- Fine's idea of the **cd**-index.
- R-labelings and Purtill's recursion for the **cd**-index of the simplex and the cube.
- Coalgebras and Newtonian coalgebras.
- **cd**-index of prisms and pyramids.
- Application to zonotopes.
- The quasi-symmetric function of a poset.
- Other topics, as time permits.

There is no textbook, but references will be given during the course.

As a part of the grade of the course, each student will present a paper in algebraic combinatorics.

Prerequisites are basic linear algebra and algebra.

New time TR $11^{40} \to 12^{55}$

New Place. Rhodes Hall 657 conference room

MATHEMATICS 651 — TOPOLOGY SEMINAR

This course — known as the Berstein Seminar — is a geometry/topology seminar in which the students do most of the lecturing and the professor tries to provide guidance without getting in the way.† When it works well it can play a significant role in a student's mathematical growth and empowerment.

For Fall 1996 I suggest* that we study and teach each other about

Metric Spaces of Non-Positive Curvature

This is a topic of much current research in geometry and geometric group theory and is, in fact the title of a book in preparation by Martin Bridson and Andre' Haefliger. Here is the opening paragraph of their introduction:

> *The purpose of this book is to describe the global properties of simply-connected spaces that are not positively curved in the sense of A. D. Alexandrov, and to examine the structure of groups which act on such spaces by isometries. Thus the central objects of study are metric spaces in which every pair of points can be joined by an arc isometric to a compact interval of the real line, and in which every triangle satisfies the CAT(0) inequality. This inequality encapsulates the concept of non-positive curvature in Riemannian geometry and allows one to faithfully reflect the same concept in a much wider setting — that of geodesic metric spaces. ... There is therefore a great deal to be said about the global structure of CAT(0) spaces, and also about the structure of groups which act on them by isometries — such is the theme of this book.*

Prerequisites: Math 551 (material on fundamental group, covering spaces, group presentations) and some elementary differential geometry (say, Math 454 or a willingness to do some collateral reading).

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† This form of the seminar was originated in 1974 by Professor Israel Berstein when he could no longer lecture traditionally due to Parkinson's disease. He taught it for the next 16 years. His quickness of mind, lively humor and warm non-judgmental support of the students have left a legacy which subsequent professors take as the model of what we should be doing in this seminar.

* Student suggestions for alternate topics are welcome and a final decision can be made on the first day of class. Please contact me ahead of time with your suggestions,if possible.

TR 8:40-9:55, WHITE [redacted] 310

ALGEBRAIC TOPOLOGY II
Math 653

MWF 10:10 -11:00 in B-29
Allen Hatcher

This is the continuation of 551. To give the course a focus, the eventual goal will be an introduction to spectral sequences, which are perhaps the most powerful tool in Algebraic Topology, as well as one of the hardest to get a feel for on first exposure.

The three main pillars at ground level in Algebraic Topology are homology, cohomology, and homotopy groups. In 551 one meets homology groups and the first homotopy group (the fundamental group), so 653 will begin with cohomology (for perhaps 4 weeks), then proceed to homotopy groups and homotopy theory generally (maybe 6 weeks), before bringing everything together in spectral sequences (the last 4 weeks).

Cohomology is a more highly-structured version of homology, with a product (cup product) which makes the cohomology groups of a space into a ring. There are still lots of open questions about cup products,even the basic one of determining which rings occur as cup product rings of spaces. Cohomology also arises naturally in various geometric contexts, such as Poincaré Duality and Obstruction Theory, both of which will be discussed somewhere in the course.

Homotopy groups are the natural higher-dimensional version of the fundamental group, but differ in being extremely hard to compute. There isn't a single finite CW complex whose higher homotopy groups are all known, except in cases where all the higher homotopy groups are zero. Spectral sequences provide a nice tool for studying homotopy groups, and near the end of the course we will prove some of Serre's theorems about homotopy groups of spheres, e.g., that they are all finite apart from a few explicit infinite cyclic factors. Spectral sequences are also a powerful tool for computing homology and cohomology groups.

This course will continue next semester as 654, though maybe not as a direct continuation. Specific topics haven't been chosen yet. One possibility is K-theory. Or perhaps some fancier homotopy theory, like rational homotopy theory and localization.

Math 661— Fall 1996
Seminar in geometry

Instructor. Reyer Sjamaar, 418 White Hall, 255-3624.

E-mail. sjamaar@math.cornell.edu.

Web. http://math.cornell.edu/˜sjamaar.

Sources. I will not use a textbook, but draw on recent journal articles and some of the following sources:

M. Audin, *The topology of torus actions on symplectic manifolds*, Birkhäuser, Boston, 1991.

N. Berline, E. Getzler, and M. Vergne, *Heat kernels and Dirac operators*, Grundlehren der mathematischen Wissenschaften, vol. 298, Springer-Verlag, Berlin-Heidelberg-New York, 1992.

V. Guillemin, *Moment maps and combinatorial invariants of Hamiltonian T^n-spaces*, Progress in Mathematics, vol. 122, Birkhäuser, Boston, 1994.

V. Guillemin and S. Sternberg, *Symplectic techniques in physics*, Cambridge Univ. Press, Cambridge, 1990, second reprint with corrections.

N. M. J. Woodhouse, *Geometric quantization*, second ed., Oxford Univ. Press, Oxford, 1992.

Keywords. Symplectic geometry and its relations to representation theory. Actions of Lie groups on symplectic manifolds. The moment map. The Darboux theorem and its generalizations. Geometric quantization and the orbit method. Equivariant cohomology and localization theorems. Symplectic quotients. Applications to toric manifolds and the Duistermaat-Heckman theorem. Symplectic cobordism.

Prerequisites. Graduate algebra and analysis, some differential topology (e. g. Math 552) and Lie theory (e. g. Math 549).

Note: time change!!

Tuesday & Thursday 8:40 – 9:55 am

Math 667 – Algebraic Geometry – Fall 1996

This course is an introduction to algebraic geometry. The main goals of this course are to develop a body of examples and geometric constructions, and to introduce many of the most important basic ideas and theorems in the subject. This should provide the necessary intuition and background for further study in algebraic geometry (such as Mark Gross' course in the spring).

Since algebraic geometry is in many respects a field built upon a wealth of examples, it is also important to be able to perform computations. Depending on the background of the class, we will cover the basics of Gröbner bases and syzygies. Throughout the course, we will attempt to compute most of the constructions which we will encounter. For this, we will use my computer algebra system, *Macaulay 2*.

Algebraic geometry is a subject that you can only learn by playing with the material, so I plan on giving weekly (do-able!) homework assignments.

The tentative course plan is:

1. First three weeks: Some basic examples (conics in the projective plane, the group law on a cubic curve, 27 lines on a cubic surface), some basic concepts (Variety, Zariski topology, Coordinate rings, regular and rational functions and maps), and some important concepts (Nullstellensatz, Bezout's theorem, blow-ups, tangent lines and singularities).
2. Gröbner bases and some initial applications. Intro to Macaulay 2.
3. Hilbert functions and Syzygies. Varieties of points.
4. After that: we will cover as much of Harris' book as seems appropriate.

Professor: Michael Stillman, White Hall 426, 255-7240

Time: MWF 12:20 – 1:10.

Location: Olin Hall 145.

Textbook: Algebraic Geometry, A first course, by Joe Harris. Hartshorne's book is also available at the bookstore, and is worth obtaining if you plan on continuing in algebraic geometry.

Prerequisites: Commutative algebra at the level of Atiyah-Macdonald would be best, otherwise you may have to either accept some results on faith, or do some extra work.

FALL 1996

Math 677 Markov Chains and Monte Carlo

This is a systematic course on the geometric theory of Markov chains along with a development of the use of Markov chains in simulation. The object of the mathematical part is to give useful bounds on time to stationarity for chains on large finite state spaces. Topics include reversible chains and eigenvalues, inequalities of Poincare, Cheeger, Sobolev, Nash and log Sobolev inequalities. I will also cover coupling, stationary times, and the Propp Wilson algorithm. Applications to metropolis and Gibbs samplers, card shuffling and computer manipulation of large finite groups. I will also present an overview of practical techniques such as Hybrid Monte Carlo, simulated tempering, and umbrella sampling which seem both useful and beyond rigorous analysis.

Prerequisites: Some exposure to probability and honest calculus. No measure theory is needed.

Persi Diaconis

8/7/96

Mathematics 686
Proof Theory

Prof. S.Artemov, fall of 1996.

The course will cover basic ideas and methods of Proof Theory along with major recent developments motivated by computer science and knowledge presentation theory. The topics will include Gentzen style and "natural" derivations, normalization theorems for classical and constructive logics, connections with the typed lambda calculus, Curry-Howard isomorphism. Arithmetization of the proof theory, Godel and Rosser incompleteness theorems, Lob theorem, reflection principles. Modal logic of the formal provability. Models of arithmetic. Consistency proofs in arithmetic and analysis and term normalization theorems in typed lambda calculus. Godel provability logic S4 and its realization in logic of proofs. Brouwer-Heyting-Kolmogorov operational semantics for the constructive logic.

NOTE: Room change!

TUE & THU 1:25–2:40

MALOTT 226