617. Dynamical systems. Instructor Yu.Ilyashenko

Basic topics. Existance and uniqueness theorems for solutions of ODE. Rectification theorem. Euler broken lines and numerical solutions of ODE. Lyapunov stability of fixed points and periodic orbits.

Attractors and horseshoes. Maximal attractors,their Hausdorff dimension and projection to planes of low dimension. Smale horseshoe and introduction to symbolic dynamics.

Elements of hyperbolic theory . Local hyperbolisity:Grobman–Hartman and Hadamard–Perron theorems. Structural stability of the Anosov diffeomorphism of torus. Expanding maps of a circle.

Dynamical systems in low dimensions. Structurally stable planar vector fields. Attractors of the planar differential equations. Diffeomorphisms of the circle: rotation number, conjugacy to the rigid rotation, density, unifirm distribution.

Introduction to KAM theory. Rapidly convergent iteration method. Diffeomorphisms of the circle close to the rigod rotation. Poincaré and Siegel linearization theorems.

Complex differential equations. Poincaré–Dulac theorem. Normal forms for Fuchsian systems. Riemann–Hilbert problem. Solvability for the plane and non-solvability for the Riemann sphere.

* * *

About 2/3 of the course will be covered by the books of Arnold "Geometric Methods in the Theory of Ordinary Differential equations" and Katok and Hasselblat,"Introduction to the Modern Theory of Dynamical Systems".

Math 652

Differential Geometry

Xavier Buff

This course will be an introduction to the theory of differentiable manifolds. The main prerequisite (which is non-negotiable) is knowledge of point set topology.

The course will cover the following topics:

- Topological manifolds, manifolds with boundary and invariance of domain theorem,
- Differentiable manifolds, tangent vector space and vector fields on a manifold,
- Exterior algebra on manifolds, exterior differentiation, integration on Riemannian manifolds,
- Regular values, Brouwer fixed point theorem, Sard's theorem,
- Oriented manifolds, the Brouwer degree, Hopf theorem, and
- Gauss-Bonnet theorem which gives the following relation between the Gaussian curvature K and Euler cahracteristic χ of a compact orientable surface M:

  $$2\pi\chi = \int_M K dA,$$

  where dA is the area element on M.

The course will make use of two books:

W. M. Boothby: An introduction to differentiable manifolds and Riemannian geometry, 2E, Ac Press.

J. W. Milnor: Topology from the differentiable viewpoint, Univ. of Virginia Press, Charlottesville, Virginia, 1965.

Fall 1997 **MW 3:05 – 4:20**

Mathematics 661

THE GEOMETRY OF SURFACES

A *geometric surface* (Euclidean, spherical or hyperbolic surface) is a metric space in which each point has a neighborhood with the geometry of a small ball in $\mathbb{E}^2$ (the Euclidean plane), $\mathbb{S}^2$ (the 2-sphere) or $\mathbb{H}^2$ (the hyperbolic plane). This course will study geometric sufaces, with an emphasis on hyperbolic surfaces. (The rest of this summary is written in terms of hyperbolic surfaces.)

Text: *The Geometry of Surfaces*, by John Stillwell (Springer Verlag 1992)

Prerequisites: Undergraduate topology and group theory (e.g., Math 453 and 434; graduate students may be taking these concurrently).

This subject matter is ideal for those beginning graduate work. The Preface of the text explains why:

> *"The theory of surfaces of constant curvature has maximal connectivity with the rest of mathematics. Such surfaces model the variants of euclidean geometry obtained by changing the parallel axiom, they are also projective geometries, Riemann surfaces and complex algebraic curves. They realize all of the topological types of compact two-dimensional manifolds. Historically, they are the source of the main concepts of complex analysis, differential geometry, topology and combinatorial group theory. (They are also the source of some hot research topics of the moment, such as fractal geometry and string theory)."*

Topics will include

- The geometry of the hyperbolic plane (especially its isometry subgroup)
- The Killing-Hopf Theorem: *Any complete, connected hyperbolic surface is of the form* $\mathbb{H}^2/\Gamma$ *where* Γ *is a discontinuous fixed point free group of isometries of* $\mathbb{H}^2$.
- *Classification of geodesics on a surface* S*: Closed geodesics correspond to free homotopy classes of loops or to conjugacy classes in the fundamental group* $\pi_1(S)$.
- *Construction of hyperbolic surfaces from polygons: (1) Any compact surface* $\mathbb{H}^2/\Gamma$ *has a fundamental polygon for* Γ. *(2) (Poincare's theorem) If a compact polygon* Π *satisfies certain side-pairing conditions and if* Γ *is the group generated by the side-pairing transformations of* Π *then* Π *is a fundamental region for* Γ.
- *Symmetric tesselations of surfaces, orbifolds, branched converings and desingularization.*

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Fall 1997 Math 711

Tuesday and Thursday 11:40-12:55

Analysis on Fractals

Jun Kigami

Fractals are used as models of shapes in nature. Hence studying physical phenomena in nature requires some kind of "Analysis on Fractals". For example, we need "Laplacians" on fractals to study waves and diffusions. The final goal of this course is to introduce the theory of "Analysis on Fractals" which has been developed in the last decade. In particular, I would like to talk about Laplacians on finitely ramified self-similar sets. Also I will introduce some basics about fractal geometry.

Part I. Fractal Geometry: construction of self-similar sets; topological properties of self-similar sets; self-similar measures; Hausdorff dimension and Box dimension; calculation of dimensions of self-similar sets

Part II. Analysis on Fractals: construction of Laplacians on finitely ramified self-similar sets; properties of harmonic functions on self-similar sets; spectral distributions of eigenvalues of Laplacians on fractals

Prerequisites: I will assume that participants are familiar with advanced calculus, basics on metric spaces and measure theory.

Mathematics 713 Course Description.

Complex Manifolds

Mark Gross

This course will be an analytic introduction to the theory of complex Kähler manifolds. The main prerequisite is familiarity with some differential geometry at the level of Volume I of Spivak; most importantly I will assume familiarity with concepts such as differential forms, de Rham cohomology, and Riemannian metrics. The topics I will cover will include:

(1) Basics of complex manifolds: definitions, differential forms on complex manifolds, Dolbeault cohomology, the de Rham and Dolbeault isomorphisms.

(2) Hermitian metrics on complex manifolds.

(3) Harmonic forms, the Hodge decomposition, and Kähler manifolds. This is the heart of the course. I plan to give a complete proof of the Hodge theorem.

(4) Applications: Kodaira vanishing and embedding theorems, Lefschetz decomposition and hyperplane theorems, examples.

(5) If there is time, I will cover one of several possible topics, possibly study of variations of Hodge structures and classifying spaces for certain complex manifolds, or Kähler-Einstein manifolds.

Math 731 now with new improved meeting times!!!

This year the algebra seminars Math 731-732 will run as a coordinated sequence in Hopf Algebras and Quantum Groups. It's not too late (9/4/97) to get in Math 731 if the sequence appeals to you.

Math 731 FALL 1997 **Seminar in Algebra: Hopf Algebras**
Prof Moss Sweedler

Coalgebras, bialgebras and Hopf algebras have application to combinatorics, algebraic groups, quantum groups and elsewhere. This will be an elementary course introducing coalgebras, bialgebras and Hopf algebras and some of the applications. The prerequisite is the introductory graduate algebra course Math 631 formerly known as 531.

Math 731 is good preparation for Math 732.

Due to scheduling difficulties the meeting times have changed. The course will meet on Tuesdays and twice on Thursdays.

Tuesday 11:30 - 12:15 .... White B25
Thursday 11:30 - 12:15 .... White B25
Thursday 2:45 - 3:45 or so ..... Baker 335

Math 732 SPRING 1998 **Seminar in Algebra: Quantum Groups**
Prof Arkady Berenstein

It would be nice if the participants of the seminar have some basic knowledge of

Algebra at the level of Math 631 (former 531)
Lie algebras and Lie groups,
Hopf Algebras (and a bit of Category Theory)

But this material will be covered or referenced as necessary.

The course will be about algebraic aspects of Quantum Groups. The most interesting aspects of Quantum Groups seem to be representations and Canonical Bases studied by Kashiwara and Lusztig.

The term "Quantum" relates this subject with quantum mechanics. Indeed, the main source of Quantum Groups are the usual Lie groups (Lie algebras) equipped with additional structures of symplectic or Poisson manifolds (Lie bialgebras). Then the term "quantization" physically means a passage from Poisson Geometry (=Clasical Mechanics) to the Noncommutative Geometry (=Quantum Mechanics). If time permits, we will go over Drinfeld's theory of quantum deformations of Poisson-Lie groups, and Lie bialgebras. The latest result in this direction is the theorem (Kazhdan and Etingof, 1995) which states that every Poisson-Lie group admits a canonical deformation-quantization.

Mathematics 735 **Fall 1997**

Representation Theory of Finite Chevalley Groups

TR 11:40–12:55pm **WE B29**

The representation theory of groups plays an important role in many branches of mathematics such as mathematical physics, harmonic analysis, topology, combinatorics and number theory, and has been an object of systematic study in itself for a long time.

A *representation* of a group G is a group homomorphism $\pi : G \to GL(V)$, where V is a vector space over some field, for this course usually $\mathbb{C}$. It is called *irreducible* if V has no proper nonzero G–invariant subspace $W \subset V$ *i.e.* there is no $0 \neq W \subset V$ such that $\pi(g)W \subset W$ for all $g \in G$.

In the case of a finite (or more generally compact) group, every irreducible representation is necessarily finite dimensional. Furthermore, every representation (finite or infinite dimensional) decomposes as a direct sum of irreducible representations, $V = \oplus V_\alpha$. The usual problems in representation theory, classification of irreducible representations, decomposition, restriction can be answered in terms of *character theory.* The character of a finite dimensional representation is the function $g \mapsto tr\pi(g)$, the usual trace of $\pi(g)$. This is a *class function,* it has the property that $tr\pi(hgh^{-1}) = tr\pi(g)$. The dimension of the space of such functions is equal to the number of conjugacy classes, *i.e.* distinct sets $\{ghg^{-1}\}$. It also equals the number of inequivalent irreducible representations; so this suggests one should try to attach a representation to each conjugacy class.

These facts are elementary and can be found in most standard algebra textbooks. But given a finite group, it is not at all easy to determine its conjugacy classes much less classify its irreducible representations. Typically one resorts to *ad hoc* methods. In addition, there is no reasonable general way to attach a representation to a conjugacy class. The finite Chevalley groups are an exception in that one can present the answer to the above questions in a systematic way. The prime example is $GL(n, \mathbb{F}_q)$, $n \times n$ matrices with nonzero determinant over a finite field with q elements. For example, determining the conjugacy classes comes down to linear algebra, namely canonical forms of matrices. There is also a systematic way to construct all the irreducible representations such that they are naturally parametrized by conjugacy classes. But to do this one needs sophisticated techniques from algebraic geometry developed by Deligne and Lusztig.

In this course I will aim to describe the character theory of the Chevalley groups. In order to keep the course as elementary as possible, we will not develop the algebraic geometry, but will simply use the properties of the constructions. This should not detract from the understanding of the material; it is the approach taken in most of the references below. In addition, the emphasis will be on $GL(n, \mathbb{F}_q)$. A student with a solid understanding of basic modern algebra and linear algebra should be able to follow the course comfortably.

REFERENCE BOOKS

R. Carter, *Finite groups of Lie type*, Wiley–Interscience, New York, 1985.
F. Digne and J. Michel, *Representations of finite groups of Lie type*, London Mathematical Society Student Texts 21.
G. Lusztig, *Characters of reductive groups over a finite field, Annals of Math. Studies*, vol. 107, Princeton University Press.
G. Lusztig, *Representations of finite Chevalley groups*, AMS, CBMS 39.

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Tu - Th

10:10 - 11:25, BARTON 301

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Mathematics 737 Course Description.

Algebraic Number Theory.

Mark Gross

This course will be an introduction to algebraic number theory at the level of *Algebraic Number Theory* by Cassels and Fröhlich. Math 531 or its equivalent is a required prerequisite, and familiarity with commutative algebra at the level of Math 534 or Atiyah and MacDonald will be very useful, especially regarding concepts such as localization, integral closure, Dedekind domains and completion. I will spend some time reviewing these notions, however.

The basic goal of algebraic number theory is to study number fields, i.e. finite extensions of the rational numbers, and number rings, i.e. the integral closure of the integers in a number field. I will cover the following topics:

(1) Review of commutative algebra in the context of number rings and fields: Dedekind domains, valuations, completion.

(2) Structure of number rings: local and global theory, prime decomposition, adeles, ideles, class groups of number rings, unit groups.

(3) Dirichlet series, L-series and the class number formula.

(4) An introduction to class field theory.

Math 751: Seminar in Topology

TR 8:40–9:55

Noel Brady

Here are some candidate topics for the seminar: Coxeter groups, Artin groups, hyperbolic groups. I am also open to suggestions for other possible topics from seminar participants. M. Davis has a nice survey article on Coxeter and Artin groups which would serve as a backbone for the course if we decide on those topics. The book by E. Ghys and P. de la Harpe (*Sur les groups hyperboliques dapres Mikhael Gromov*, Prog. in Math. vol. 83, Birkhauser, 1990) is a good source for hyperbolic groups. If people have questions or suggestions before August 21, they should email me at brady@math.cornell.edu

MATH 753

Algebraic Topology II

MWF 10:10-11, White 310

This will be a basic course in algebraic topology beyond the first semester. We will cover topics in homology, cohomology, and homotopy theory. Specifically, if time permits, we will study homology with coefficients, the Künneth formula, cohomology, universal coefficient theorem, cup products and the ring structure of cohomology, Poincaré duality, homotopy groups, Whitehead's Theorem, Hurewicz's Theorem, Eilenberg-MacLane spaces, fibrations and fiber bundles, and obstruction theory.

Possibly, we can touch lightly other topics such as cohomology operations and stable homotopy groups.

The text will be Hatcher's remarkably readable *Algebraic Topology* I, which will be available from the department.

J. West

Mathematics 757

Fall 1997

Analysis, geometry, and probability on finitely generated groups

The starting point for this course is Harry Kesten's thesis (published in 1959), which initiated the systematic study of random walk on finitely generated infinite groups. [Start at the identity of the group and repeatedly multiply by a randomly chosen generator or its inverse.] Following Kesten, we will be interested in invariants of a group G that can be derived from the behavior of the random walk, and we will try to relate these invariants to the algebraic structure of G. Following Varopoulos, we will widen the scope of the investigation by bringing in geometric and analytic ideas, such as isoperimetric inequalities and Nash–Sobolev inequalities. This turns out to be crucial in order to settle some of the fundamental questions in the subject. Sample question (Kesten, 1966): Which groups are recurrent, i.e., for which groups will the random walker eventually return to his starting point with probability 1? Answer (Varopoulos, 1986): A group is recurrent if and only if it is virtually abelian of rank at most 2. Less formally, the only recurrent groups are the ones discovered by Polya in 1920.

The original papers of Varopoulos require a substantial background in real analysis. But I will make use of some notes and papers by Saloff–Coste and his collaborators, which make the subject accessible to people with a much more modest background. The only prerequisites for the course are Math 511 and undergraduate algebra (no probability required). Given these prerequisites, the course will be mostly self-contained, except for an occasional need to quote a more advanced result without proof.

Ken Brown

Course description for Math 761, Fall 1997

In this course, we shall primarily study the heat kernel on a Riemannian manifold from both the analytical and geometrical point of view. We will attempt to cover some major results obtained during the last two decades concerning the estimates of heat kernel using various techniques such as gradient estimates, Moser iteration and comparison principle. At the same time, we will try to indicate some of the applications to geometry and topology. Depending on the background of the audience, we may first cover some facts involved from the Riemannian geometry.

The following is a list of the topics to be covered.

1) Existence and some basic properties of heat kernel
2) Gradient estimates and sharp bounds of heat kernel
3) Moser iteration and parabolic Harnack inequality
4) Characterization of parabolic Harnack inequality
5) Comparison principle and heat kernel on algebraic varieties

Note: The class meets on WF from 3:45 → 5:00 PM at B29 instead of TTH 8:40 – 9:55 AM

Jiaping Wang

Math 777 --- Probability problems that arise from genetics.

Rick Durrett --- rtd1@cornell.edu

DESCRIPTION ----- The course will begin by following Claudia Neuhauser's notes from Math 8690, a course she taught at Minnesota in the Fall of 1996:

1+2. Biological background and basic models
4. The coalescent and the ancestral selection graph
5. Neutral theory versus selectionist view
6. Sampling theory in the neutral case.
7. Sampling theory in the presence of selection.
8. Spatial Models
9. Population models in varying environments.

The discussion will be supplemented by material from some of Claudia's sources, as well as from more recent papers.

PREREQUISITES ----- Familiarity with discrete or continuous time Markov chains is necessary, but may not be sufficient.

TIME ----- 2:30 MWF in White Hall B29

FIRST MEETING ----- Monday, September 1

Copies of Claudia's notes will be available on 9/1