617. DYNAMICAL SYSTEMS. INSTRUCTOR YU. ILYASHENKO

Basic topics.

Existance and uniqueness theorem for solutions of ODE. Rectification theorem. Euler broken lines and numerical solutions of ODE. Lyapunov stability of fixed points and periodic orbits.

Attractors and horseshoes.

Maximal attractors, their Hausdorff dimension and projection to planes of low dimension. Smale horseshoe and introduction to symbolic dynamics.

Elements of hyperbolic theory.

Local hyperbolicity: Grobman-Hartman and Hadamard-Perron theorems. Structural stability of the Anosov diffeomorphism of a torus.

Dynamical systems in low dimensions.

Structurally stable planar vector fields. Attractors of the planar differential equations. Diffeomorphisms of the circle: rotation number, conjugacy to the rigid rotation, density, unifirm distribution.

Introduction to KAM theory.

Rapidly converging iteration method. Diffeomorphisms of the circle close to the rigid rotation. Poincaré and Siegel linearization theorems.

Complex differential equations.

Poincaré-Dulac theorem. Normal forms for Fuchsian systems. Riemann-Hilbert problem. Solvability for the plane and nonsolvability for the Riemann sphere.

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About 2/3 of the course will be covered by the books of Arnold "Geometric Methods in the Theory of Ordinary Differential equations" and Katok and Hasselblat, "Introduction to the Modern Theory of Dinamical Systems".

Math 619

Partial Differential Equations

TR 10:10-11:25A   WE 328

Instructor: José F. Escobar

Text: Partial Differential Equations by Lawrence C. Evans, Graduate Studies in Math., Vol 19, AMS.

This is a basic graduate course on the basic theory of partial differential equations. I will follow the new textbook closely. In the first semester we will cover most topics in Part I and Part II of Evan's book including a study of the transport, Laplace, heat and wave equations, non–linear first order PDE's, Sobolev spaces (Sobolev and Poincaré inequalities) and second order elliptic equations. If time permits we will study linear evolution equations.

PLEASE NOTE THE NEW SCHEDULE

Math 631 Information
Fall 1998

Math 631, formerly 531, is the introductory graduate algebra course. A prerequisite for it is an undergradate modern algebra course such as Math 434. Also a knowledge of linear algebra at the level of Math 433 will be assumed, though not much direct use will be made of it. The material falls naturally into three parts with, however, many interconnections. A partial list of topics follows:

Group Theory: Subgroups, normal subgroups, quotient groups, group actions, Sylow theorems, composition series.

Rings and Modules: Subrings, ideals, quotient rings, principal ideal domains, modules, fundamental theorem on finitely generated modules over a PID, application to abelian groups, tensor products.

Fields: Field extensions, Galois theory, application to the theory of equations (solvability by radicals etc.).

Note that there is a considerable overlap with Math 434 but that the pace will be much faster during the "review" portions of the course.

Text: None officially required, but Lang's "Algebra" will be on reserve in the Math library. Many other books, for example those by Herstein or Jacobson, could be suitable references.

Grading: Grades will be based on weekly homework assignments, a take-home midterm exam and a take-home final.

My office hours: Monday and Friday, 11 - 12, in White 210, and by appointment.

Shankar Sen

Math 649, 1998

LIE GROUPS and LIE ALGEBRAS

Instructor: E. B. DYNKIN

Tuesday and Thursday, 11:40–12:55. B-25 White Hall. The first lecture on August 27

This is an introduction to the theory of Lie groups and algebras and their linear representations — a fundamental part of many branches of Mathematics (algebra, differential and algebraic geometry, topology, harmonic analysis, differential equations...) and an important tool in modern Physics (elementary particles, gauge theory, strings...). Only a basic knowledge of mathematical analysis and linear algebra is required. Elements of theory of differentiable manifolds will be introduced as needed.

I shall try to avoid generalities and technicalities and to emphasise ideas illustrated on concrete examples.

The following topics will be covered.

1. The groups of real and complex matrices and their classical subgroups. The corresponding Lie algebras. Exponential mapping.
2. Groups of smooth transformations of a differentiable manifold. The corresponding Lie algebras of linear differential operators. Invariant differential operators of higher order.
3. General concept of a Lie algebra. Construction of the corresponding Lie group via the Campbell-Hausdorff formula.
4. Structure of semisimple Lie algebras. Root systems. Simple roots.
5. Linear representations of semisimple groups. Description of an irreducible representation by the highest weight. A tensor construction. Weyl's character formula.
6. Semisimple subalgebras of semisimple Lie algebras. Application to classification of primitive transformation groups. The representation of a subalgebra induced by a representation of an algebra.

Math 652
Differential Manifolds

White B-29, T Th 1:25 - 2:40

David Henderson
White B-28, dwh2@cornell.edu

Prerequisites: Undergraduate linear algebra, and undergraduate analysis with some topology

Text: Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*

We will cover most of Chapters I-IV of Boothby and parts of Chapter V, including the topics:

topological manifolds
smooth manifolds
immersions and embeddings
tangent bundles
vector fields
transformation groups and Lie groups acting on a manifold
quotient manifolds and covering manifolds
bilinear forms and Riemannian metrics

[Much of the rest of the text will be covered in Math 653.]

Along the way we will apply what we learn to discussions of the possible global topology and geometry of our physical universe, thought of as a smooth 3-dimensional manifold (we will not treat relativity). Including (as much as we can):

1. We will look at the 8 possible local (homogeneous) geometries for 3-manifolds, and discuss why only the Euclidean, Spherical, or Hyperbolic 3-dimensional geometries are likely as the local geometry of our universe.

2. In the next few years two space probes, NASA's Microwave Anisotropy Probe (2000-2001) and ESA's Planck satellite (about 2003) will map the background microwave radiation and then (if the universe is sufficiently globally curved) these maps will record interference patterns which will determine (within bounds) the global topology and geometry of the universe. We will discuss how this determination can be done.

Math 661
Discrete Geometry, Distance Geometry and Rigid Structures
Fall 1998
R. Connelly

This is an introduction to the geometry of points and distances with applications to and from the theory of rigid and non-rigid structures. A basic role of geometry in science and mathematics is to determine when distance constraints on a configuration of points determine the configuration itself. This is connected to the theory of frameworks as used in engineering and well as distance geometry in mathematics. A brief list of topics that I hope to cover during the semester is described below. (The original content of this course as Geometric Topology has been changed for just the Fall of 1998. We will still do geometry, and a little topology may creep in but that is all.)

Prerequisites: A good background in linear algebra (including matrices, determinants, symmetric matrices, eigen vectors, etc.) and some basics of calculus. A little abstract algebra incuding the definintion of a finite group would help, but it is not necessary.

Topics: (The unfamiliar words below will be defined in the course. The following is meant to suggest the flavor of what is to be covered.)

1. A classification of the congruences of Euclidean space.
2. Infinitesimal and static rigidity of frameworks and tensegrities
3. Infinitesimal rigidity implies rigidity
4. Stresses and spider webs
5. Applications to glasses and protein structure
6. Cauchy's Theorem abut the rigidity of convex polyhedra
7. The stress-energy quadratic form/mathix
8. Super stability and global rigidity
9. Applications to the packing of congruent spherical balls
10. The calculation of highly symmetric tensegrities using representations of finite groups

More information and links: The word tensegrity was coined by R. Buckminster Fuller to describe a structure that was created by Kenneth Snelson, a sculptor. These are structures made of sticks that are suspended in mid-air with cables attached at the ends or the sticks. The questions as to what geometric properties determine its stability are a major subject of this course. See the brief introduction to tensegrities. For a catalogue of pictures of several hundred different examples of tensegrities that are stable enough to be built, see the catalogue constructed with Allen Back, and a general introduction, "Mathematics and Tensegrity", in the March-April 1998 issue of the American Scientist. For an application of the idea of a tensegrity to biology as a way of understanding the structure of the cell, see the article "The Architecture of Life", by Donald Ingber, in the January 1998 issue of the Scientific American.

Instructor:

R. Connelly
124 White Hall
Phone: 255-9928
e-mail: connelly@math.cornell.edu

Meeting times and room:

Tuesday-Thursday, 2:55 to 4:10 PM, in White B29

Homework:

There will be regular weekly homework assignments, a take-home final and at least one take-home prelim.

MATH 671, FALL 1998
INSTRUCTOR: H.KESTEN

The textbook for the course is

R.Durrett, Probability:

Theory and Examples.

This book will be followed rather loosely only. The principal topics to be discussed in the course are:

Probability spaces, Sample spaces and random variables, Distribution functions, Expectation, Independence, Borel-Cantelli lemma, Zero-one laws, Convergence of random variables, Characteristic functions and inversion theorem, Laws of large numbers and central limit theorem. If time perits we shall start on Markov chains.

H. Kesten

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Attention! time change

Math 711 **Analysis seminar** **Fall 1998**

Sobolev type inequalities and some of their applications

This course will focus on different aspects and applications of certain functional inequalities, the most famous of which are Sobolev inequalities. Sobolev inequalities control the size of a compactly supported smooth function in terms of the size of its derivatives. They assert that

$$\|f\|_q \leq C\|\nabla f\|_p$$

for all $f \in \mathcal{C}_0^\infty(\mathbb{R}^n)$ where $q = np/(n-p)$ and $1 \leq p < n$.

We will start by discussing Sobolev imbedding and Sobolev and Poincaré inequalities in Euclidean space. As an application, Moser's proof of Harnack inequality for uniformly elliptic second order differential operator in divergence form will be presented.

Moving to the more general setting of Riemannian manifolds, we will discuss the equivalence between a number of Sobolev type inequalities, including Nash and Faber-Krahn inequalities. We will relate these inequalities to spectral properties. In particular we will discuss the Rozenblum-Cwikel-Lieb inequality for the number of negative eigenvalues of Schrödinger operators of the form $-\Delta - V$.

In a different but related direction we will see how Sobolev inequalities relate to volume growth and isoperimetry as well as to uniform and Gaussian heat kernel upper-bounds. We will characterize in terms of Poincaré inequalities and volume growth those Riemannian manifolds which satisfy a uniform Parabolic Harnack inequality.

We will also discuss the use of Sobolev type inequalities in the context of semigroups of operators.

Some references:

E. Stein. *Singular Integrals and Differentiabilty Properties of Functions*, 1970, Princeton University Press.

D. Gilbarg, N. Trudinger. *Elliptic Partial Differential Equations of Second Order*, 1977, Springer.

W. Maz'ja. *Sobolev Spaces*, 1985, Springer.

E.B. Davies. *Heat Kernels and Spectral Theory*, 1989, Cambridge University Press.

N. Varopoulos et al. *Geometry and Analysis on Groups*, 1993, Cambridge University Press.

~~10.10-11.25 WE 310~~

The class will meet each Tuesday and Thursday, ~~11.40-12.55~~ in WE 310

First meeting Tuesday September 1 1.10 328

Laurent Saloff-Coste
lsc@math.cornell.edu

COURSE SYLLABUS

Mathematics 717 and T&AM 776: Applied Dynamical Systems
Fall 1998

John Guckenheimer
White Hall 310
Tuesday-Thursday 1:25-2:40

Prerequisites: Good courses in undergraduate analysis, multivariable calculus and linear algebra. Some exposure to ordinary differential equations or dynamical systems will be helpful.

Nonlinear dynamical systems are used as models in every field of science and engineering. Universal patterns of behavior, including "chaos," have been observed in large sets of examples. Mathematical theories describing geometrically the qualitative behavior of "generic" systems explain many of these patterns. This course will discuss dynamical systems theory and its application to examples. Several representative examples from different disciplines will be described at the beginning of the course and used throughout the semester to illustrate theoretical ideas. Emphasis will be placed upon bifurcation, the qualitative changes in solutions that occur as system parameters are varied. Computational methods for the analysis of dynamical systems will also be discussed. Both the performance of algorithms and their mathematical foundations will be considered. Further development of these computational methods is an active research area, and the course lectures will repeatedly deal with this frontier. Computer laboratory sessions will be held in addition to lectures.

Requirements: Homework and a student selected project.

MATH 731: Topics in Group Theory

Fall, 1998

Keith Dennis
White 320
Telephone: 255-4027
e-mail: dennis@math.cornell.edu

This is a course for those who have already had an introductory abstract algebra course, such as one based on Herstein. Topics will include groups acting on sets, Sylow theorems and generalizations, p-groups, nilpotent, solvable, supersolvable groups, group extensions, Schur-Zassenhaus theorem, permutation groups, wreath products, ...

One of the main themes of the course will be "classification" – determine all groups having certain properties or the orders of groups which have certain properties. E.g., such questions as the following: Determine for which integers n every group of order n is abelian, nilpotent, solvable, supersolvable, ... Determine all groups for which every subgroup is normal. Determine all groups which have a fixed-point free action on a vector space.

Although this course will have some topics in common with Math 635 given by Diaconis, it should contain sufficient different material to retain the interest of those who took that course.

The course meets ~~Tuesday/Thursday 11:30–12:55 in White 328.~~ The first meeting will be on ~~Tuesday, September 1~~.

MWF 3:35 - 4:25 White 324

Monday, Aug. 31

Math 737 NUMBER THEORY

I will give an introduction into the theory of modular forms. This is special class of functions on the upper half plane which are of significance in analytic number theory; for example, modular forms played a role in the Wiles proof of the Fermat's conjecture. We will discuss their L–functions and applications of modular forms to counting the number of integral solutions of quadratic equations. In the last part we show that modular functions can be considered as functions on the general linear group GL(2,$\mathbf{R}$) which are invariant under GL(2,$\mathbf{Z}$) and which satisfy a differential equation.

This course the first part of a year long introduction to the classical and modern theory of automorphic forms. The second part in spring will be taught by Prof. Ramakrishna and can be taken independently of this course.

Birgit Speh

The first meeting of this course is on Monday August 31

Math 751: Seminar in Topology

~~Organizational meeting - Thursday 8/27, 8:40-9:55 am, White Hall, Room 310~~

Meeting Monday, Wednesday ~~at 3 PM~~ 10 AM in Room 233 (J. West's office

This week only: WF 10AM

We will focus on papers that employ elementary topological methods to acheive interesting group theoretical results. We will especially keep an eye towards 3-manifold groups. The lecturing will be done by participants.Perhaps a week or two will be devoted to each topic.

Possible topics include:

The Loop Theorem and Dehn's Lemma.
Peter Scott's proof of the coherence of 3-manifold groups.
Recent work of Feighn and Handel on the coherence of mapping tori of free group endomophisms.
Stallings' paper on the topology of graphs. Perhaps followed by some papers on the Hanna Neumann conjecture.
Peter Scott's proof that surface groups and certain 3-manifold groups are subgroup separable.
Rubinstein-Wang's paper on pi_1-injective surfaces in graph manifolds.
A survey on graphs of groups and graphs of spaces?
A survey of small-cancellation theory or word-hyperbolic groups?

Dani Wise
499d White Hall
daniwise@math.cornell.edu

Math 753
ALGEBRAIC TOPOLOGY

MWF 10:10-11:00
B25 White Hall

This is the second semester of Algebraic Topology, the sequel to Math 651. After a brief review of homology, we will introduce cohomology with its ring structure, prove the universal coefficient theorem, and study Poincaré duality. We will then proceed to higher homotopy groups and fibrations, and the Whitehead and Hurewicz theorems. We will build Eilenberg-MacLane spaces and Postnikov towers, and introduce the idea of an obstruction theory.

Karen Vogtmann

Math 757

l_2-homology and Coxeter groups

Fall 1998
Boris Okun

The first part of this course is an elementary introduction to l_2-homology theory. I will give basic definitions, explain how to make Betti numbers l_2 and develop analogs of the usual machinery of algebraic topology (long exact sequences, Mayer-Vietoris sequences, Poincare duality etc.).
In the second part I plan to describe recent work with Mike Davis, calculating l_2-homology groups of (some) right -angled Coxeter groups. In particular I plan to give a proof of Hopf conjecture for cubical manifolds of dimension 4 and explain various other conjectures and relations between them.

This course should be accessible to any person with good background in algebraic topology, I hope to keep the analytical part to the minimum.

Time Change:

TR 11:40 WE 310

Math 777 **Fall 1998** **R. Durrett**

Stochastic Calculus: A Practical Introduction

MWF 1:25-2:15

My aim is to cover the first 6-8 chapters of my book with same name (CRC Press).

I will begin with a brief treatment of *Brownian motion* (Chapter 1). The next topic is *stochastic integration* (Chapter 2). There are many details involved in the precise definition of the integral, all of which are spelled out in the book. Thus I simply will describe the main steps in the construction of the integral and give the main ideas of the proofs of the most important formulas, i.e., those that make it into the Chapter Summary on pages 79-81.

Once the machinery of stochastic calculus is developed, it becomes possible to derive a number of *facts about Brownian motion* (Chapter 3) and about *partial differential equations* (Chapter 4). Our focus will then shift to the solution of *stochastic differential equations* (Chapter 5), paying special attention to the *one dimensional diffusions* (Chapter 6) for which it is possible to do a lot of explicit calculations. If all goes well we will end with a result on convergence of Markov chains to limiting diffusions (Section 8.7). For this to be intelligible we will have to give a brief treatment of *semigroups and generators* (Chapter 7) and *weak convergence of stochastic processes* (Chapter 8).

Along the way we will add some material from mathematical finance, not in the current version of the book: option pricing by absence of arbitrage in discrete markets, the Black Scholes model, replication of contingent claims (a.k.a., completeness of the Brownian filtration). In the section on diffusion processes, we will consider the Cox Ingersol Ross, and Heath Jarrow Morton interest rate models. People looking to get rich quick should be warned that this material will only be about 10-15% of the course. We will also be interested in applications of stochastic calculus to biology (Feller's continuous branching process, diffusion processes in genetics) and to mathematics itself (harmonic functions and other solutions of PDE's).

Comments and questions can be directed to: rtd1@cornell.edu. The time slot has been chosen carefully to avoid conflicts with the CAM and statistics colloquia, and with the class that I teach at 10:10 MWF. Only a Titanic tragedy would inspire me to change the meeting time of the class.

MATH 787
SET THEORY
FALL 1998

This course will be a basic introduction to axiomatic set theory beginning with the axioms for Zermelo-Fraenkel Set theory and the elementary theory of ordinal and cardinal numbers. We will develop enough of the structure of Gödel's constructible universe L to prove the consistency of the general continuum hypothesis, the axiom of choice and various combinatorial principles useful for establishing consistency results in topology and algebra (e. g. the Souslin and Whitehead problems). We will also investigate some of the forcing constructions of Cohen, Martin, Solovay and others to construct models of set theory in which the continuum hypothesis fails and various problems of combinatorics, topology and algebra have different solutions than they do in Gödel's universe. There may also be some discussion of combinatorial properties of some of what are now considered to be the smaller of the large cardinals.

Prerequisites: A familiarity with predicate logic and naive set theory.
Text: *Set Theory, An Introduction to Independence Proofs*, K. Kunen

Richard A. Shore

Fall 1998
Math 788

Topics in Applied Logic

Instructor: Richard Platek
Time: Mondays 4:30 – 6:30 PM
Place: White Hall B25
Topic: System and Software Safety

Many systems which interact with their environments have the potential of causing harm. Historically Safety Engineering emerged as a discipline which rationally analyzed possible hazards and designed mechanisms to reduce operational risks. The recent rapid transition to software intensive systems is having a major impact within Safety Engineering. In this course we will explore how Mathematical Logic is providing powerful tools for managing the complexity of life critical systems. Within Computer Science this area is called Formal Methods.

The course will include:

- Case studies of system failures (e.g., the Therac 25 software controlled medical instrument which is responsible for the death of several people – Software Kills)
- Reviews of successful applications of Formal Methods to increase safety (e.g., the Paris metro)
- Examples of the use of software to replace traditional mechanisms and the risks involved (e.g., commercial fly by wire airplanes: Airbus 320 and Boeing 777)
- Commercially available Formal Methods tools
- The research perspective