520 Partial Differential Equations   Spring 1997

Instructor. Vladimir Veselov

Textbook. Text: Robert McOwen, Partial Differential Equations, Prentice Hall, 1996.

Course Description. This is a second course on the theory of partial differential equations (PDE) and a natural sequel of Math 519.

It will not assume any prior familiarity with PDE (however, some familarity with the classical methods for solving the Laplace equation and the heat equation under the usual boundary conditions would be helpfull) but assumes a background in Analysis and knowledge of the very basic concepts of Banach and Hilbert spaces. In the first part of this course we will develop the theory of elliptic equations, centering especially on the solution of the Dirichlet problem. The theory of weak derivatives will be developed, various inequalities and imbedding theorems will be proven. In the second part the theory of semigroups will be developed. We will use semigroups of operators on a Banach space to describe the dynamics of evolutionary PDE. In particulary, we will solve the initial-boundary problem for parabolic equations.

Math 532: NONCOMMUTATIVE ALGEBRA
Spring 1997

In the modern algebra the Ring Theory is of a great importance. Historically, the rings emerged from various branches of mathematics: Number theory (rings of integers, orders, Hecke rings), Representation theory (group rings, enveloping algebras, rings of matrices), Differential Geometry and Functional analysis (Algebras of functions and operators, e.g. differential operators), Homological Algebra (cohomology rings, abelian categories, Grothendieck K-groups).

One of the main the goals of the course is to give a systematic introduction to the noncommutative Ring Theory, and to emphasize the role of this theory in Algebra (and, if possible, in Geometry).

I will mostly follow the textbook by Farb and Dennis, *Noncommutative Algebra*. More precisely, I will cover the first four chapters of the text thoroughly: semisimple modules and rings, the Wedderburn structure theorem, the Jacobson radical, central simple algebras, and the Brauer group. I then plan to emphasize the following additional topics as time permits:

(1) Group cohomology and the cohomological interpretation of the Brauer group

(2) Representation theory of finite groups.

Other reference books:

S. Lang, *Algebra* Addison-Wesley, 1993

T. Lam, *A first course in noncommutative rings* Springer-Verlag, 1991

K. Goodearl, R. Warfield, Jr *An introduction to noncommutative noetherian rings* New York, 1989

The prerequisite for this course is a knowledge of commutative and abstract algebra at the level of Math 531.

Arkady Berenstein

420 White Hall

255–4959

Mathematics 551, Algebraic Topology

Text: Algebraic Topology, by Allen E. Hatcher (to be obtained from the mathematics department).

Topics to be studied include the following:
1. Fundamental group and covering spaces, including the relation between subgroups of the fundamental group of X and covering spaces of X.
2. CW complexes, an especially good class of spaces on which to apply algebraic topology.
3. Homology theory. This will include singular homology theory (for any spaces) and cellular homology theory for CW complexes.
4. Cohomology theory. A little homological algebra, showing how to pass from a chain complex to cohomology groups, a discussion of categories and functors, the relation between homology and cohomology, and universal coefficient theorems. Finally, the first evidence for studying cohomology, the cup product, which turns the cohomology groups of a space into a graded ring.
5. There are many exercises included in the book, several of which will be assigned. Some of these will give good examples of the applications of algebraic topology to other areas of mathematics

G.R.Livesay

DIFFERENTIABLE MANIFOLDS II

MW 3:05 – 4:20 .... White B25

This is the second semester in the Math 552–553 sequence. It will be taught independently of Math 552.

Prerequisites: A basic course in differentiable manifolds (atlasses, tangent bundles, vector fields, Lie brackets, etc.) but not Riemannian manifolds. Also, for Parts II and III of the course, some elementary algebraic topology (as in Math 551) will be assumed.

PART I: A Short Course in Riemannian Manifolds (3–4 weeks)

Starting with the definition of a Riemannian manifold, we will study topics concerning volumes, connections, geodesics and curvature in Riemannian manifolds. The lectures will roughly be based on selections from Chapters V–VIII of Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry.*

Part II: Introduction to Morse Theory (3 weeks)

We will read Part I of Milnor's book, *Morse Theory* (Annals of Math Studies # 51 , Princeton University Press, 1969).

Part III: The h-Cobordism Theorem (7 weeks)

We will read Milnor's book *The h-Cobordism Theorem* (Princeton University Press, 1965). This treatment is unconventional in that it proceeds by modifying a Morse function on an h-cobordism until a new Morse function is achieved which has no critical points. The view of this process as one of modifying a handlebody structure — introduced by Smale who first proved this theorem and much used in geometric topology — will also be discussed.

If time permits the *s-cobordism theorem* which is a central generalization of the h-cobordism theorem to non-simply connected spaces will also be discussed.

In Parts II and III of the course, students will be asked to lecture on some of the sections in the books of Milnor alluded to above.

Marshall Cohen (White 229, 5-2392, marshall@math.cornell.edu)

Math 572: Probability Theory

Spring, 1997

Math 572 is the second semester course in a one-year graduate sequence course on Probability Theory. We will continue using Rick Durrett's *"Probability: Theory and Examples, Second Edition "* as textbook. The topics of M572 will include

Markov chains: Markov property, recurrence and transience, stationary distribution, convergence theorem;

Birkhoff's ergodic theorems, mixing and entropy;

Brownian motions: construction and basic properties, strong Markov properties, Blumenthal's 0-1 law, maxima and zeros of Brownian path;

Skorokhod's theorem on embedding random walk into Brownian motion, Central limit theorems for martingales and for dependent variables;

Empirical distributions and Brownian bridge;

Laws of the iterated logarithm for Brownian motion and random walks with finite variance.

The prerequisite for this course is a knowledge of Math 571 or the equivalent.

Other Reference Books

P. Billingsley: Probability and Measures. 1979.

Leo Breiman: Probability. 1968.

William Feller: An Introduction to Probability theory and its Applications. Vol. I, third edition (1968); Vol II, second edition (1971).

Zhen-Qing Chen

SPRING 1997, ROGER FARRELL

A one semester course to cover some basics of the mathematics used in point estimation and hypothesis testing. References will be made to the two books by Eric Lehmann, Tests of Statistical Hypotheses (TSH) and Theory of Point Estimation (TPE), and other sources on convexity. Purchase of TSH is optional, TPE should be purchased in the preprint form (revision) which will be produced at gnomon copy.

Prerequisites: Math 571 and the equivalent of Math 413-414.

Topics that may be covered include:

Review of integration, some distribution theory, some matrix theory.
Groups and invariance.
Exponential families, monotone likelihood ratios, standard examples.
Sufficient statistics, factorization, randomized estimators.
Complete statistics, a theorem of Basu.
Convex functions, Jensen's inequality, convex loss.
Unbiased estimation, linear models, UMVU estimation.
The order statistic, nonparametric estimation, examples.
Cramer-Rao-Blackwell inequality. Fisher information.
Proof of admissibility in some standard examples.
Maximum likelihood estimation, counter examples, asymptotic theory.
Finite dimensional convex sets, separation, the minimax theorem.
Weak* compactness (see TSH appendix).
Tests: the set of alpha-beta points. Power functions.
UMP, unbiased, invariant tests.
Randomization, two ways of randomization.
Decision theory, Bayes, minimax.
Two stage and sequential tests, estimation.

MAPLE: It is possible to do interesting computations here, particularly of an asymptotic type. I may try this again.

January 12, 1997

MATHEMATICS 581: LOGIC

Spring Semester 1997 TR 1:25-2:40

The course is a graduate level introduction to first order logic. No previous knowledge of logic is necessary, though some familiarity with formal systems is useful. The topics include the syntax and semantics of first-order languages, predicate calculus, completeness and compactness, recursive functions and incompleteness theorems, resolution and logic programming, and other topics if time permits.

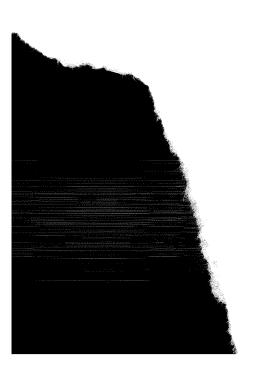Text: H.-D. Ebbinghaus, J. Flum, W. Thomas: Mathematical Logic

Miklós Erdélyi-Szabó

Math 612   Wavelets   Spring 1997

<u>Instructor</u>: Yang Wang (visiting Professor from Georgia Tech)

<u>Text</u>: "A first course on wavelets" by Hernandez and Weiss, CRC Press 1996.

This course focuses on the fundamental mathematical properties of wavelets. We start from multi–resolution analyses and show how wavelets can be constructed from them. Fundamental properties such as orthogonality, smoothness, and self–similarity of wavelets will be discussed thoroughly. In the mean time, we shall also discuss other important topics related to wavelets such as uncertainty principles, frames, self–affine tiles, as well as wavelets not constructed from multi-resolution analyses. Part of the course will be devoted applications (such as filter banks) and research topics.

MATHEMATICS 622
TOPICS IN COMPLEX ANALYSIS
Riemann Surfaces
SPRING 1997

This course is an introduction to Riemann surfaces, quasiconformal mappings and Teichmuller theory. We will cover the following topics:

1. Riemann surfaces
   - Compact Riemann surfaces
   - Conformal mappings
   - Riemann-Roch theorem
   - Fuchsian groups

2. Deformation theory
   - Quasiconformal mappings
   - Teichmuller space
   - Complex structure
   - Extremality
   - Beltrami equation
   - Boundary dilatation

3. Quadratic differentials
   - Teichmuller's mappings
   - Infinitesimal theory

4. Some applications in Dynamics
   - Holomorphic motions and vector fields on closed sets
   - Teichmuller spaces of the complement of a Cantor set

References:
   - H. M. Farkas & I. Kra, Riemann surfaces
   - L. V. Ahlfors, Lectures on quasiconformal mappings.
   - F. P. Gardiner, Teichmuller theory and quadratic differentials.
   - Y. Imayoshi & M. Taniguchi, An introduction to Teichmuller spaces

Nikola Lakic
Nikola@math.cornell.edu

Math 632

Arithmetic Groups.

I plan to give an introduction to give an introduction to the theory of arithmetic groups Γ. Roughly speaking an arithmetic group is discrete subgroup of a group G defined in some way to related to the arithmetic structure of a field K. The easiest example of such a group are the integers $\mathbf{Z} \subset \mathbf{R}^{+}$. Less trivially, the ring of integers of an arbitrary number field K can be viewed as a discrete subgroup of the additive group $R^{n}(n = [K : Q])$. A basic problem is to find a set of coset representatives for G/Γ having decent properties. This frequently has interesting arithmetic interpretation, which we will discuss in special cases.

In the last part of the course I will give an introduction to the cohomology of arithmetic groups.

We will treat in this course only certain concrete examples in order to avoid excessive prerequisites although the methods are quite general.

Literature: James Humphreys, Arithmetic Groups, Springer Lecture Notes 789
A. Borel, Introduction aux groupes arithmetiques, Hermann, Paris 1969
G. Harder, Cohomology of arithmetic groups, mimeographed notes

Prerequisites I will assume only basic knowledge of algebra and analysis. Familiarity with Lie groups and some number theory would be helpful.

Birgit Speh

A Second Course in Group Theory: Math 635
SPRING 1997

This is a course in finite group theory for people who have had an introduction at the level of Herstein. Topics to be covered include p-groups(1): Sylows Theorems, Burnside Basis Theorem, groups of order p^3 and p^4. Nilpotent groups: upper and lower central series, supersolvable groups. Commutator calculus p-groups(2): cannonical forms; regular p-groups, an introduction to Lie ring methods.

Free groups. Generators and relations. Extensions of groups and cohomology: introduction to Schur Zassewhaus theory. Permutation groups: multiple transitivity, Jordans Theorem, Mathiu groups and an introduction to the classification.

The course will have a combinatorial flavor: computing and bounding diameters, counting the number of generating sets, learning our way around the permutation group and some Chevally groups.

Course meets Tuesday/Thursday 11:40–12:55.

Persi Diaconis

Mathematics 639 Course Description.

Algebraic Geometry

Mark Gross

This course will be a sequel to Mike Stillman's fall 1996 Algebraic Geometry course. The text will be *Algebraic Geometry* by Robin Hartshorne.

The main goal of the course will be to cover the modern language of algebraic geometry. This is the language of schemes, and is covered in chapters II and III of Hartshorne's book. However, I also hope to get to some interesting applications which demonstrate the importance of this language. In particular, by the end of the course, we should be able to prove the Weil conjectures for counting points on curves over finite fields.

The topics covered will be roughly the following:

(1) Sheaves.

(2) The definition of schemes and various properties of schemes.

(3) Weil and Cartier divisors, projective embeddings of schemes.

(4) Differentials.

(5) Cohomology of sheaves and Serre duality.

(6) Geometry of curves and surfaces.

(7) Curves over finite fields.

As prerequisites, a strong commutative algebra background is necessary, and I will assume some familiarity with homological algebra, but a willingness to accept statements about homological algebra is sufficient.

Math 640 -- Homological Algebra

Professor: Mike Stillman

Time: 8:40 - 9:55 Tuesday, Thursday

Location: Uris Hall 204

This course will introduce the important techniques of homological algebra. Homological algebra is not a subject in its own right, but serves as an important tool in many fields, including commutative algebra, algebraic geometry, algebraic topology, and many other fields. What makes homological algebra particularly powerful is that it may be used in a very abstract way, obtaining results quite quickly, but on the other hand, it is very computational. Unfortunately, this material is often taught only at the abstract level, and the insight and computational power is often missed. In the first part of this course, we will provide this computational viewpoint. Students will use my computer algebra system Macaulay 2 to compute and manipulate complexes and homology groups.

A tentative list of topics includes

Motivation and examples
Simplicial homology. Free resolutions of ideals.
Complexes
The basic method of diagram-chasing, and a computational view of this. (5-lemma, snake lemma, connecting homomorphism, long exact sequence, operations on complexes, etc)
Projective and injective modules
We will attempt to understand these rather than just know the universal properties that characterize them.
Tensor products and Hom
Again, intuition and their meaning in specific examples will be stressed. (The key abstract facts will also be given!)
Projective and injective resolutions
Tor and Ext
Key by-products of homological algebra, and what gives the subject much of its power.
Group Cohomology
An introduction, with examples. Hilbert theorem 90.
Categories and functors
The language of categories and functors.
Derived functors
Tor and Ext done more systematically.
Spectral sequences
A key tool, but one has to overcome the morass of book-keeping notation.

Other topics may include: Abelian categories, more group cohomology, cyclic cohomology, hypercohomology, and/or an introduction to the derived category.

Prerequisites: Math 531, and a desire to learn this powerful tool.
Textbook: An introduction to homological algebra, by Charles Weibel.

Math 652

BERSTEIN SEMINAR IN TOPOLOGY
TuTh 2:55-4:10

The aim of this course is to work through a fundamental book, paper or sequence of papers in topology. Topics should be accessible to anyone who has completed the usual first-year sequence of graduate courses, including Math 551. The course will be run as a seminar, i.e. students present the material in informal talks to the rest of the class, and questions and discussion are actively encouraged. The specific topic will be decided upon at the first class meeting. My own bias is toward topics which involve the connections between topology, geometry and group theory, such as the theory of groups acting on trees, the Nielsen-Thurston theory of surfaces and mapping class groups, hyperbolic and CAT(0) spaces and groups, isoperimetric inequalities in groups, and automatic structures on groups. However, I would be happy to discuss other potential topics with students at any time.

Karen Vogtmann

Math 654. Algebraic Topology III

~~T&Th 11:40-12:55 in room B-25~~ Mon & Fri 1:25-2:40 in B-29

This will be an introduction to some more advanced topics in Algebraic Topology centered around generalized cohomology theories, specifically:

1. Topological K-theory. This is something of a toy model (but powerful in its own way!) for the later cohomology theories. One could spend the better part of a semester developing K-theory fully, but I intend only to sketch the highlights, e.g., not proving Bott periodicity, which is the foundation of K-theory.

2. Cobordism, particularly complex cobordism. This has been the most powerful cohomology theory for applications to homotopy theory for the past 20 years or more. Unlike K-theory, however, a fair bit of algebraic topology machinery, for example the Adams spectral sequence, is needed just to understand the basic structure of complex cobordism, so we shall spend some time setting up this machinery.

3. Variants of complex cobordism: the localized-at-a-prime version called BP (for Brown-Peterson, not British Petroleum) and the further refinements of BP called Morava K-theories.

4. Elliptic cohomology. This is a hot topic which people are excited about because of its connections with things outside topology ("elliptic" refers to elliptic curves in algebraic geometry and number theory). It would be nice to see at least what the definition is, though I doubt we will have time to really study it seriously.

<div align="center">Allen Hatcher</div>

Math 658   Topics in Topology   TR 10:10 'til 11:25

Instructor: John Meier

Text: None

Topic: Finiteness properties and Σ-invariants of infinite groups

Description: If you want to gain some control over an infinite group, you need to have some sort of 'finiteness' present in the group. The most common finiteness property is finite generation, and the next most common is finite presentation. Using the intuition developed from these two properties one can develop a few different notions of "higher dimensional" finiteness properties. This course will begin by examining standard facts about finite generation and presentation, and then move on to the higher dimensional analogues. Along the way we will learn how to define the 'dimension' of an infinite group, and how to build canonical topological spaces ($K(\pi,1)$'s) for groups.

We will also examine the question of finiteness properties of subgroups of a given group. For example, if H is a finite index subgroup of a finitely generated group G, then H is itself finitely generated. This is not always true if H is of infinite index. For example, if φ is *any* map from a free group F_n onto $\mathbb{Z}$, then the kernel of φ is not finitely generated. In the second half of the course we will study techniques which describe the finiteness properties of infinite index subgroups of a given group G.

Specific topics we will cover should have non-trivial intersection with the following list:

- Eilenberg-MacLane spaces for a group, and the $\mathcal{F}_m$ and FP_m finiteness properties.
- The geometric and homological dimensions of an infinite group.
- Finiteness properties and group extensions.
- The Σ^1-invariants of Bieri, Neumann and Strebel. An emphasis will be placed on concrete computations of these invariants.
- The higher Σ-invariants.
- The recent examples of Bestvina and Brady showing that the FP_m and $\mathcal{F}_m$ finiteness properties are distinct.
- The recent work of Bieri and Geoghegan on the finiteness properties of kernels of actions on non-positively curved spaces, assuming we have the time and they get it written up!

Prerequisite: The pace of the course will depend greatly on the background and interests of the people attending. I won't expect people to know homological group theory, but folks should have experienced covering spaces, homology and homotopy at the level of Math 551.

Mathematics 662, Spring 1997

Differential Geometry

Instructor: John Hubbard.

This course will cover the following topics:

1. Connections and curvature

2. De Rham cohomology and Charactersitic classes

3. Global analysis (Banach manifolds ...)

4. Donaldson-Seiberg-Witten theory

References.

For connections, I will provide notes. Volume 2 of Spivak's *Differential geometry* is also a very good source. I advocate buying the entire series (Mike Spivak is a friend of mine).

For characteristic classes, Milnor's notes give a beautiful treatment. The book by Bott and Tu is also a good source, as is volume 5 of Spivak.

For Global analysis, Lang's book on differential manifolds is one possible source, but I do not much like the presentation. I will see what else I can find.

For Donaldson-Seiberg-Witten theory, Morgan's book is the obvious reference.

Prerequisites.

I will assume that the audience knows Differentiable manifolds, Differential forms, Stokes theorem, and the definition and first properties of Vector bundles. Math 551, taken at least concurrently, will be very helpful.

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

Math 678

PROBABILITY AND ANALYSIS: THEOREMS AND PROBLEMS

E. B. Dynkin

Tuesday and Thursday, 1:25–2:40

Probabilistic methods in analysis are based on integration over functional spaces. On the other hand, probabilistic models describing physical phenomena, like the random motion of a particle (the random walk, the Brownian motion) or a cloud of particles (branching particle systems, the super-Brownian motion) provide an intuitive insight into many analytical problems.

We select topics where, as a result of interaction between probabilists and analysts, a substantial progress has been made recently and where many challenging problems remain still open. Among these topics are boundary-value problems for linear and quasi-linear elliptic differential equations; linear and nonlinear potential theory, connections between capacities (Choquet, Riesz, Bessel, Poisson, Martin ...) and hitting probabilities by random closed sets; positive solutions of the equations $\Delta u = 0$ and $\Delta u = u^\alpha$ ...

We address pure and applied mathematicians interested in new trends in stochastic analysis. The emphasis will be made on ideas rather than on technicalities. No preliminary knowledge of probability theory is required, and Math 511 (or the first eight chapters of Rudin's *Real and Complex Analysis*) provides a sufficient background in analysis.

January 12, 1997

MATHEMATICS 683: MODEL THEORY

Spring Semester 1997 TR 10:10-11:25

The course is an introduction to model theory. I will assume some previous knowledge of logic (MATH 581 is more than enough); some familiarity with algebra and basic set theory is also useful, but not required. Topics include elementary equivalence and elementary substructures, automorphisms, interpretations, preservation theorems, introduction to stability, countable models, model completeness, Feferman-Vaught theorem, saturation; nonstandard methods and categoricity if time permits.

Text: Wilfrid Hodges: Model Theory

Miklós Erdélyi-Szabó